

Today, I will delve into one of the most important topics (ubiquitous application) that are tested on GMAT. It is also one of the topics that will appear time and again during MBA e.g. in Corporate finance, you might be taught how to find 'Weighted Average Cost of Capital'. So it will be highly beneficial if you have a feel for weighted average concepts.

The first question is – What is Weighted Average? Let me explain with an example.

A boy's age is 17 years and a girl's age is 20 years. What is their average age?

Simple enough, isn't it? Average age =  $(17 + 20)/2 = 18.5$

It is the number that lies in the middle of 17 and 20. (Another method of arriving at this number would be to find the difference between them, 3, and divide it into 2 equal parts, 1.5 each. Now add 1.5 to the smaller number, 17, to get the average age of 18.5 years. Or subtract 1.5 from the greater number, 20, to get the average age of 18.5 years. But I digress. I will take averages later since it is just a special case of weighted averages.)

Now let me change the question a little.

There are 10 boys and 20 girls in a group. Average age of boys is 17 years and average age of girls is 20 years. What is the average age of the group?

Many people will be able to arrive at the following:

Average Age =  $(17*10 + 20*20)/(10 + 20) = 19$  years

Average age will be total number of years in the age of everyone in the group divided by total number of people in the group. Since the average age of boys is 17, so total number of years in the 10 boys' ages is  $17*10$ . Since the average age of girls is 20, the total number of years in the 20 girls' ages is  $20*20$ . The total number of boys and girls is  $10 + 20$ . Hence you use the expression given above to find the average age. I hope we are good up till now.

To establish a general formula, let me restate this question using variables and then we will just plug in the variables in place of the actual numbers above (Yes, it is opposite of what you would normally do when you have the formula and you plug in numbers. Our aim here is to deduce a generic formula from a specific example because the calculation above is intuitive to many of you but the formula is a little intimidating.)

There are  $w_1$  boys and  $w_2$  girls in a group. Average age of boys is  $A_1$  years and average age of girls is  $A_2$  years. What is the average age of the group?

Average Age =  $(A_1*w_1 + A_2*w_2)/(w_1 + w_2)$

This is weighted average. Here we are not finding the average age of 1 boy and 1 girl. Instead we are finding the average age of 10 boys and 20 girls so their average age will not be 18.5 years. Boys have been given less weightage in the calculation of average because there are only 10 boys as compared to 20 girls. So the average has been found after accounting for the weightage (or 'importance' in regular English) given to boys and girls depending on how many boys and how many girls there are. Notice that the weighted average is 19 years which is closer to the average age of girls than to the average age of boys. This is because there are more girls so they 'pull' the average towards their own age i.e. 20 years.

Now that you know what weighted average is and also that you always knew the weighted average formula intuitively, let's move on to making things easier for you (Tougher, you say? Actually, once people know the scale method that I am going to discuss right now (It has been discussed in our Statistics and Problem Solving book too), they just love it!)

So, Average Age,  $A_{avg} = (A_1*w_1 + A_2*w_2)/(w_1 + w_2)$

Now if we re-arrange this formula, we get,  **$w_1/w_2 = (A_2 - A_{avg})/(A_{avg} - A_1)$**

So we have got the ratio of weights  $w_1$  and  $w_2$  (the number of boys and the number of girls). How does it help us?

Knowing this ratio, we can directly get the answer. Another example will make this clear.

John pays 30% tax and Ingrid pays 40% tax. Their combined tax rate is 37%. If John's gross salary is \$54,000, what is Ingrid's gross salary?

Here, we have the tax rate of John and Ingrid and their average tax rate.  $A_1 = 30\%$ ,  $A_2 = 40\%$  and  $A_{avg} = 37\%$ . The weights are their gross salaries – \$54,000 for John and  $w_2$  for Ingrid. From here on, there are two ways to find the answer. Either plug in the values in the formula above or use the scale method. We will take a look at both.

1. Plug in the formula

$w_1/w_2 = (A_2 - A_{avg})/(A_{avg} - A_1) = (40 - 37)/(37 - 30) = 3/7$

Since  $A_1$  is John's tax rate and  $A_2$  is Ingrid's tax rate,  $w_1$  is John's salary and  $w_2$  is Ingrid's salary

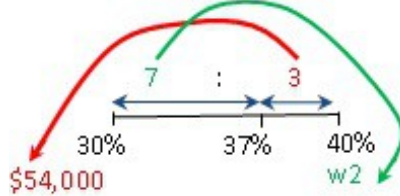
$w_1/w_2 = \text{John's Salary}/\text{Ingrid's Salary} = 3/7 = 54,000/\text{Ingrid's Salary}$

So Ingrid's Salary = \$126,000

It should be obvious that either John or Ingrid could be A1 (and the other would be A2). For ease, it a good idea to denote the larger number as A2 and the smaller as A1 (even if you do the other way around, you will still get the same answer)

## 2. Scale Method

On the number line, put the smaller number on the left side and the greater number on the right side (since it is intuitive that way). Put the average in the middle.



The distance between 30 and 37 is 7 and the distance between 37 and 40 is 3 so  $w_1:w_2 = 3:7$  (As seen by the formula, the ratio is flipped).

Since  $w_1 = 54,000$ ,  $w_2$  will be 126,000

So Ingrid's salary is \$126,000.

This method is especially useful when you have the average and need to find the ratio of weights. Check out next week's post for some 700 level examples of weighted average.